1. Remark: $\int_0^\infty p(x)e^{-|ax|}$ for all polynomials $p(x)$ and coefficients a is defined.

2. Examples (applications of Criterion 1):

2.1. (1) $\int_1^\infty \frac{\sin x}{x} dx$

We see $\frac{|\sin x|}{x} \leq \frac{1}{x}$ but harmonic series isn't useful. We integrate by parts, so $\int_1^b \frac{\sin x}{x} dx = -\int_1^b -\frac{1}{x^2} \cos x dx = -\frac{1}{b}\cos b + \cos 1 - \int_1^b \frac{1}{x^2}\cos x dx$. LHS limit exists iff $-\int_1^b \frac{1}{x^2}\cos x dx$ limit exists. But clearly $-\int_1^b \frac{\cos x}{x^2} dx$ is bounded by $-\int_1^b \frac{1}{x^2}$ whose limit exists.

2.2. (2) $\int_{\frac{-\pi}{2}}^\infty \tan x dx$

2.3. (3) $\int_0^{\pi/2} \frac{1}{\sqrt{\sin x}} dx$

Sketch: clearly for large x limit exists, we only concern ourselves with small x. We know $\lim_{x\to 0} \frac{\sin x}{x} = 1$ and $\sin x \geq \frac{1}{2}x$ for x small. Since $\int_0^{\pi/2} \sqrt{2}x^{-1/2} dx$ limit exists we can prove our desired result.

2.4. (4) $\int_0^1 \ln x dx$

3. Criterion 2: If $f(x) > g(x) \geq 0$ and $\int_a^\infty g(x)dx$ doesn't exist then $\int_a^\infty f(x)dx$ doesn't exist.